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Quarterly Progress Report TSVE-QPR-1
September - December, 1952

# RESEARCH ON THERMAL STRESSES AND STRAINS IN MEDIA EXHIBITING TEMPERATURE-DEPENDENT VISCOELASTIC PROPERTIES

Contract No. AF 33(616)-291
E. O. No. R-461-1 BR-1

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University of Illinois
Aeronautical Engineering Department

Prepared by
Harry H. Hilton
Harry H. Hilton
Project Director

December 10, 1952

Approved by
H. S. Stillwell, Head
Aeronautical Engineering Department

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# INTRODUCTION

Numerous excellent solutions exist for thermal stresses and strains in classical elastic bodies, but unfortunately these cannot take into account either the actual influence of the temperature on the material properties or the flow phenomena that real materials do exhibit at elevated temperatures.

Zener's theoretical analysis and Kê's experimental verifications have demonstrated that metals do not obey Hooke's law at any level of stress, although in the low stress ranges the strains are still completely recoverable. This inelastic behavior is called anelastic or viscoelastic.

The author has presented solutions for plane thermal stress and plane thermal strain problems for viscoelastic bodies of the Maxwell type. These solutions incorporate not only flow phenomena occurring at elevated temperatures, but also take into account the temperature dependence of the material properties.

It is the purpose of this research project to extend the analysis of thermal stresses to more complex viscoelastic media (such as Kelvin bodies and the standard linear solid) and at the same time include actual temperature variations of the material properties.

It is planned to conduct a systematic investigation of thermal stresses in plates and cylinders under various temperature gradients in order to obtain an over-all picture of the behavior of viscoelastic materials under these conditions.

## PROGRESS

In the first three months of this project four separate studies have been initiated, namely

(1) Analysis of thermal stresses in a thick wall cylinder under the influence of a steady state radial temperature gradient for viscoelastic materials of
   (a) the Kelvin type
   (b) the standard linear solid type

(2) Analysis of thermal stress in a thin circular plate under the influence of a steady state radial temperature gradient for viscoelastic materials of
   (a) the Kelvin type
   (b) the standard linear solid type

In each case it is planned to solve the problem in the most general manner so that the over-all picture may be obtained. Numerical examples for specific temperature gradients will be computed to illustrate the results.

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## PARTICIPATING PERSONNEL

During the period September 15 to December 14, 1952 the following persons worked on this project on a part-time basis:

David G. Herbener, Undergraduate Computer
Dwight A. Mahaffy, Undergraduate Computer
Bernard K. Genetti, Research Assistant
Ernest B. Wolansky, Research Assistant
Harry H. Hilton, Assistant Professor, Project Director

## TOTAL EXPENDITURES

Total estimated expenditures for the period September 15 to December 14, 1952 are $1,630.

Since these are the beginning months of the project, the expenditures for this period could only be estimated. However, it is believed that the estimated amount is reasonably accurate.

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December 1952 - March 1953

# RESEARCH ON THERMAL STRESSES AND STRAINS IN MEDIA EXHIBITING TEMPERATURE-DEPENDENT VISCOELASTIC PROPERTIES

Contract No. AF 33(616)-291

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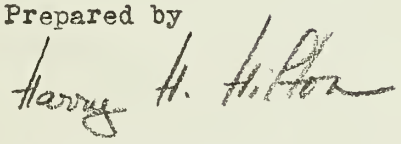
December 1952 - March 1953

# RESEARCH ON THERMAL STRESSES AND STRAINS IN MEDIA EXHIBITING TEMPERATURE-DEPENDENT VISCOELASTIC PROPERTIES

Contract No. AF 33(616)-291
E. O. No. R-461-1 BR-1

University of Illinois
Aeronautical Engineering Department

Prepared by



Harry H. Hilton
Project Director

March 10, 1953

Approved by

H. S. Stillwell, Head
Department of Aeronautical Engineering

## INTRODUCTION

It is the purpose of this research project to carry out a systematic investigation into the problem of thermal stresses in viscoelastic media. The stress-strain relationships that are used describe actual phenomena much more exactly than present day elastic solutions, since

(a) The material properties are taken as temperature dependent

(b) The viscoelastic or anelastic time effects are included in the formulation of the problem.

At present, there exists an elastic-viscoelastic analogy* for homogeneous viscoelastic bodies not subject to temperature gradients. It is hoped that enough insight will be gained from the four distinct problems under investigation to generalize Alfrey's analogy to the thermal stress problem in viscoelastic media with temperature dependent properties.

A first approximation of this extension has been accomplished during the last month and will be briefly described subsequently in this report. A detailed technical report is being prepared and will be submitted shortly.

---

* T. Alfrey, "Non-homogeneous stress in viscoelastic media", Quart. Jour. Appl. Math, vol. 2, pp. 113-119, 1944.

## LIST OF SYMBOLS

$E_{ij}$ = strain deviator

$G$ = shear modulus

$S_{ij}$ = stress deviator

$t$ = time coordinate

$T$ = temperature

$x_i$ = cartesian coordinate ($i = 1,2,3$)

$\alpha$ = coefficient of thermal expansion

$\epsilon$ = mean strain

$\eta$ = coefficient of viscosity

$\sigma_{ij}$ = stress component

$\tau_\epsilon$ = relaxation time at constant strain

$\tau_\sigma$ = relaxation time at constant stress

## THE VISCOELASTIC STRESS-STRAIN RELATIONSHIP

The general viscoelastic stress-strain relation for an incompressible medium, may be written as

$$\epsilon_{ij} = \alpha T \tag{1}$$

$$P\{S_{ij}\} = 2Q\{E_{ij}\} \tag{2}$$

where the differential operators $P$ and $Q$ define the particular viscoelastic medium.

For the three cases under investigation, the operators are

(a) Maxwell body

$$P = \frac{1}{G}\frac{\partial}{\partial t} + \frac{1}{\eta} \tag{3}$$

$$Q = \frac{\partial}{\partial t} \tag{4}$$

(b) Kelvin body

$$P = 1 \tag{5}$$

$$Q = G + \eta\frac{\partial}{\partial t} \tag{6}$$

(c) Standard linear solid

$$P = 1 + \tau_\epsilon \frac{\partial}{\partial t} \tag{7}$$

$$Q = G\left[1 + \tau_\sigma \frac{\partial}{\partial t}\right] \tag{8}$$

It must be remembered that the material properties, i.e. $G$, $\alpha$, $\eta$, $\tau_\epsilon$ and $\tau_\sigma$, are all considered to be temperature dependent, making the medium inherently non-homogeneous.

## STATEMENT OF THE PROBLEM

Currently, there are four distinct analyses under investigation. They deal with the determination of thermal stresses in viscoelastic media with temperature dependent properties of the Kelvin and standard linear solid types in:

(a) a thick-walled circular cylinder under the influence of a steady-state radial temperature gradient

(b) a thin circular plate subjected to a steady-state radial temperature gradient.

The present status of this research is as follows:

(1) Analysis (a) for the Kelvin body has been completed and numerical examples are being computed. A technical report covering this phase of the program is being prepared and will be submitted as soon as the computations are carried out.

(2) In analysis (a) for the standard linear the governing differential equation of the problem has been solved. In order to evaluate the arbitrary functions resulting from the integration of the differential equation, it is necessary to solve an integral equation. Work is progressing to evaluate this integral equation in such manner that a general solution to this problem may be presented for any arbitrary steady-state temperature gradient.

(3) In analysis (b) for the Kelvin body and standard linear solid, the differential equations have been formulated but have not yet been solved.

The general mode of attack in any of these four analysis is to solve simultaneously the equations of equilibrium and compatibility together with the stress-strain

relationship. In each of these analysis it has been possible to reduce these simultaneous equations to a single differential equation. The procedures involved will appear in the technical reports now in preparation and it is thought that any presentation here would only duplicate the future reports.

## EXTENSION OF ALFREY'S ELASTIC-VISCOELASTIC ANALOGY TO THERMAL STRESS PROBLEMS

In the past month, Alfrey's elastic-viscoelastic analogy has been extended to cover the thermal stress problem. This extension is, however, only a first approximation to the problem since it has been assumed that the material properties are temperature independent. These results will be of importance since they will serve as a comparison to the results of identical problems where the temperature effects on material properties are included.

The results may be briefly stated as follows:
For the general viscoelastic material obeying the stress-strain laws given by Eqs. (1) and (2), and subject to a temperature distribution

$$T(x_i, t) = H(t)\, F(x_i) \tag{9}$$

where $H$ and $F$ are respectively functions of $t$ and $x_i$ only, the thermal stresses are of the form

$$\sigma_{ij}(x_i, t) = g(t)\, \sigma^{*}_{ij}(x_i) \tag{10}$$

The symbols $\sigma^{*}_{ij}(x_i)$ are the equivalent elastic stresses caused by a steady state temperature distribution $F(x_i)$. The response function $g(t)$ is found from the differential equation

$$P\{g(t)\} = Q\{E(t)\} \tag{11}$$

This means that for any viscoelastic thermal stress problem, where the temperature dependence of the material properties is neglected, it is possible to use the existing elastic thermal solutions and modify them with the response function as indicated by Eq. (10).

## TECHNICAL REPORTS IN PROGRESS

The following two technical reports are being prepared:

TSVE-TR-1 "Thermal Stresses in Thick-Walled Cylinders Exhibiting Temperature Dependent Viscoelastic Properties of the Kelvin Type"

TSVE-TR-2 "An Extension of Alfrey's Elastic-Viscoelastic Analogy to Viscoelastic Thermal Stress Problems"

These reports will be submitted as soon as they are completed.

## PARTICIPATING PERSONNEL

During the period February 16 to March 15, 1953 the following persons worked on this project on a part-time basis:

Hassan A. Hassan, Research Assistant

Hal G. Russell, Research Assistant

Ernest B. Wolansky, Research Assistant

Harry H. Hilton, Assistant Professor, Project Director

## TOTAL EXPENDITURES

Total expenditures for the period September 15, 1952 to January 31, 1953 were $3,031.52. Estimated expenditures from January 31, 1953 to March 14, 1953 were $750.00

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# RESEARCH ON THERMAL STRESSES AND STRAINS IN MEDIA EXHIBITING TEMPERATURE-DEPENDENT VISCOELASTIC PROPERTIES

Contract No. AF 33(616)-291
E.O. No. R-461-1 BR-1

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# RESEARCH ON THERMAL STRESSES AND STRAINS IN MEDIA EXHIBITING TEMPERATURE-DEPENDENT VISCOELASTIC PROPERTIES

Contract No. AF 33(616)-291
E.O. No. R-461-1 BR-1

University of Illinois
Aeronautical Engineering Department

Prepared by

Harry H. Hilton
Project Director

June 8, 1953

Approved by

H. S. Stillwell, Head
Aeronautical Engineering Department

# STATUS OF RESEARCH

It is the purpose of the work under the research contract AF 33(616)-291 to formulate a rational method of analysis for the determination of thermal stresses in media having temperature-dependent viscoelastic properties.

With this aim in mind a number of idealized problems have been treated in order to obtain an overall picture of the general problem. The problems so far considered have been:

(1) A circular thick-walled cylinder with a steady-state radial temperature distribution

(2) A circular thin plate with a steady-state radial temperature distribution

(3) A thin-walled circular cylinder with a steady-state axial temperature distribution

In each case viscoelastic stress-strain relationships of the Maxwell, Kelvin and standard linear solid are considered. These three bodies typify the behavior of viscoelastic materials. These idealized problems have in themselves direct applications. For instance, with proper boundary conditions, case (1) is applicable to heated pressure vessel, case (2) to a turbine disc and case (3) represents an exhaust passage of a jet engine.

The procedure used in solving these problems is to formulate the equations of equilibrium, compatibility and stress-strain and then solve them simultaneously. Considerable time has been spent to obtain solutions to these differential equations in closed form.

In some cases this has been successfully accomplished, in other cases infinite series solutions had to be used. In all cases, the solution presented are of a general type and can be used for any arbitrary temperature distribution.

The following technical reports are being prepared describing the results of completed research:

TSVE-TR-1 "Thermal Stresses in Thick-Walled Cylinders Exhibiting Temperature-Dependent Viscoelastic Properties of the Kelvin Type."

TSVE-TR-3 "Thermal Stresses in Thick-Walled Cylinders Having Temperature-Dependent Standard Linear Solid Properties."

TSVE-TR-4 "Thermal Stresses in Circular Plates with Viscoelastic Temperature-Dependent Properties of the Kelvin Type."

TSVE-TR-5 "Thermal Stresses in Circular Plates with Temperature-Dependent Standard Linear Solid Properties."

The numerical examples that are being computed to illustrate these solutions are holding up these reports. Numerical integrations have to be performed and they are proving to be time consuming.

The following technical report has been completed and submitted:

TSVE-TR-2 "An Extension of Alfrey's Elastic-Viscoelastic Analogy to Viscoelastic Thermal Stress Problems".

The problem thermal stresses in a thin-walled cylinder due to an axial temperature distribution is being studied and a preliminary solution for the Kelvin body has been obtained.

## PLANS

It is planned to complete the four reports indicated and submit them as they become available. The problem of the thin-walled cylinder will be carried to completion for the Kelvin and Maxwell bodies and the standard linear solid.

The next problem to be treated will be that of a plate with a temperature distribution through its thickness. After these problems are completed an attempt will be made to generalize and extend this solution in order to obtain some form of an anology for temperature-dependent viscoelastic media such as the one described in report TSVE-TR-2.

Beginning on June 15, 1953 all personnel will devote their full-time to this research work and it is expected that more results will be available shortly.

## PERSONNEL

During the period May 14 to June 15, 1953 the following persons worked on this project on a part-time basis:

Donald E. Henning, Undergraduate Computer

Hassan A. Hassan, Research Assistant

Hal G. Russell, Research Assistant

Harry H. Hilton, Assistant Professor, Project Director

## TOTAL EXPENDITURES

Total expenditures for the period September 15, 1952 to April 30, 1953 were $5,259.03. Estimated expenditures from May 1, 1953 to June 15, 1953 were $900.00

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